J. M. Montesinos-Amilibia

Abstract. It is announced that the Freudenthal compactification of an open, connected, oriented 3-manifold is a 3-fold branched covering of $S^3$. The branching set is as nice as can be expected. Some applications are given.

Resumen. La compactificación de Freudenthal de una 3-variedad abierta conexa y orientable es una cubierta de 3 hojas ramificada sobre $S^3$. La ramificación es tan simple como podría esperarse. Se ofrecen algunos corolarios.

Open 3-manifolds as 3-fold branched coverings

Theorem 1 Let $M$ be an open, connected, oriented 3-manifold. Let $\widehat{M}$ denote its Freudenthal compactification. Then, there exist a 3-fold branched covering $p : \widehat{M} \to S^3$ such that $p$ maps the end space $E(M)$ of $M$ homeomorphically onto a tame subset $T$ of $S^3$. The 3-fold branched covering $p|_{M} : M \to S^3 - T$ is simple, and the branching set is a locally finite disjoint union of strings (properly embedded arcs).

This Theorem generalizes the Theorem of Hilden ([6] and[7]) and the author ([12]and[13]).

Corollary 1 Let $M$ be an open, connected, oriented 3-manifold with just one end. Then there exist a 3-fold covering onto Euclidean 3-space $p : M \to \mathbb{R}^3$, branched upon a locally finite disjoint union of strings.

This is the case of the uncountably many open, contractible 3-manifolds.

Corollary 2 Every closed, oriented 3-manifold is a 3-fold covering of $S^3$ branched over a wild knot.
Corollary 3 The open contractible Whitehead manifold is a 2-fold covering of \( \mathbb{R}^3 \) branched over a string.

Smith proved in [15] that for any orientation-preserving involution of \( S^3 \), the fixed point set is either null, the 1-sphere, or \( S^3 \) itself. Montgomery and Zippin [14], following pioneering work by Bing [1], showed that there is an involution in \( S^3 \) whose fixed point knot is not the unknot, since it contains a Cantor set whose complement is not simply connected. The following Corollary qualifies that result.

Corollary 4 There exist a 2-fold branched covering \( p : S^3 \to S^3 \) defined by an orientation-preserving involution \( \sigma \) of \( S^3 \) whose fixed point knot contains a non tamely embedded Cantor set. Therefore \( \sigma \) is not equivalent to the standard involution. Moreover \( p \) sends that Cantor set homeomorphically upon a tamely embedded Cantor set.

References


