RACSAM
Rev. R. Acad. Cien. Serie A. Mat.
VOL. **101** (1), 2007, pp. 45–50
Análisis Matemático / Mathematical Analysis

Enlargements of operators between locally convex spaces

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Abstract. In this note we study three operators which are canonically associated with a given linear and continuous operator between locally convex spaces. These operators are defined using the spaces of bounded sequences and null sequences. We investigate the relation between them and the original operator concerning properties, like being surjective or a homomorphism.

Extensiones de Operadores entre Espacios Localmente Convexos

Resumen. En esta nota estudiamos tres operadores que están asociados canónicamente a un operador lineal y continuo previamente prefijado entre espacios localmente convexos. Estos operadores se definen utilizando los espacios de sucesiones acotadas y de sucesiones nulas. Investigamos la relación existente entre ellos y el operador original con respecto a ciertas propiedades, como ser sobreyectivo o un homomorfismo.

1 Introduction, notation and preliminaries

Let E, F be locally convex spaces, l.c.s., and let $\mathcal{U}_0(E)$ denote the family of all closed absolutely convex zero neighbourhoods in E. We consider $\ell_\infty(E)$, the space of bounded sequences in E, endowed with the topology given by the zero neighbourhood basis $U^{\mathbb{N}} \cap \ell_\infty(E)$, $U \in \mathcal{U}_0(E)$. We also consider $c_0(E)$, the space of null sequences, with the topology inherited from $\ell_\infty(E)$, and their quotient. Let $L(E,F)$ be the set of linear and continuous operators from E to F. Given $T \in L(E,F)$ we consider the associated operators T^∞, T^0 and $T^\star$ defined as follows:

$$\begin{array}{cccc} T^\infty: & \ell_\infty(E) & \longrightarrow & \ell_\infty(F) \\ & (x_n)_n & \longrightarrow & (Tx_n)_n, \\ T^0: & c_0(E) & \longrightarrow & c_0(F) \\ & (x_n)_n & \longrightarrow & (Tx_n)_n, \\ T^\star: & \ell_\infty(E)/c_0(E) & \longrightarrow & \ell_\infty(F)/c_0(F) \\ & (x_n)_n + c_0(E) & \longrightarrow & (Tx_n)_n + c_0(F) \end{array}$$

These operators are well-defined, linear and continuous. The space $\ell_\infty(E)/c_0(E)$ is called an *enlargement* of E, since E can be identified with the quotient of the set of convergent sequences in E. Harte used the foregoing operators to generalize some results on the invertibility of operators from the frame of Banach spaces to normed spaces [12]. This was possible because $\ell_\infty(E)/c_0(E)$ is complete, whenever E is normed [12, Th. 4.5.2]. Let us recall some of these results:

Presentado por José Bonet.
Recibido: 9 de junio de 2006. *Aceptado*: 11 de Octubre de 2006.
Palabras clave / Keywords: Continuous linear operators, Fréchet spaces, (DF)-spaces, homomorphisms, spaces of bounded sequences.
Mathematics Subject Classifications: (2000) Primary 46A04; Secondary 46A45, 46Z40, 47B37

Theorem 1 ([12, Th. 3.3.5.2 & Th. 3.4.5.2]) *Let X, Y be normed spaces $T \in L(X,Y)$.*

1. *If $T^\star$ is injective, then T is a monomorphism, and if T is a monomorphism, then $T^\star$ is also.*
2. *$T^\star$ is almost open if and only if T is almost open.*

An operator $T \in L(E,F)$ is a *homomorphism* if for every $U \in \mathcal{U}_0(E)$ there exists $V \in \mathcal{U}_0(F)$ such that $V \cap T(E) \subset T(U)$. An injective homomorphism is a *monomorphism*, also called a *bounded below operator*. A surjective homomorphism $T \in L(E,F)$ is an *open* operator. The behaviour of these operators between normed spaces has been studied by Berberian, Harte, and Abramovich et al. [1, 2, 12]. T is said to be *almost open* if for every $U \in \mathcal{U}_0(E)$ there exists $V \in \mathcal{U}_0(F)$ such that $V \subset \overline{T(U)}^F$. Open and almost open operators coincide when E, F are Fréchet spaces by the open mapping theorem, see e.g. [15, p. 166] or [17, Lemma 8.2]. Besides, the almost open operators coincide with the nearly open operators in the sense of Pták with dense range, see [16, p. 24]. For a detailed study of almost open operators between normed spaces see [12]. Bonet and the author studied the topological properties of the set of monomorphisms and almost open operators in [6]. The role of duality between these classes of operators was treated in [5].

In this note we study several properties of the operators T^∞, T^0 and $T^\star$ in the setting of locally convex spaces, in which new phenomena appear, as our examples show.

2 Injectivity

Let $T \in L(E,F)$. Clearly, T^∞ and T^0 are injective whenever T is. The converse is also true looking at the first coordinate. Clearly, if $T^\star$ is injective, then T is injective. However, the injectivity of T is not enough to conclude the converse as the next trivial example shows:

Example 1 *Consider the space of all summable sequences ℓ_1, let $(e_n)_n$ denote the canonical basis and $T \in L(\ell^1)$ be defined as $Te_n := 1/n^2 e_n, n \in \mathbb{N}$. We have $(1/n^2 e_n)_n \in c_0(\ell_1)$, but $(e_n)_n \notin c_0(\ell_1)$. Therefore $T^\star$ is not injective because $\ker(T^\star)$ is not trivial.*

Proposition 1 *Let $T \in L(E,F)$ be injective. Suppose that one of the following conditions is verified:*

(1) *E is semi-Montel, or*

(2) *$T^{-1} : T(E) \subset F \to E$ is sequentially continuous,*

then $T^\star$ is injective.

PROOF. To proof (1) fix $(x_n)_n \in \ell_\infty(E)$ such that $(Tx_n)_n \in c_0(F)$. Suppose on the contrary that $(x_n)_n \notin c_0(E)$, then there exists $U \in \mathcal{U}_0(E)$ and a subsequence $(x_{n_k})_k$ such that $x_{n_k} \notin U, k \in \mathbb{N}$. Since $(x_{n_k})_k$ is bounded, there exists an adherent point to it, say $x \in E$. Therefore Tx is adherent to $(Tx_{n_k})_k$ and $Tx = 0$. So $(x_{n_k})_k$ has 0 as an adherent point, a contradiction. On the other hand, (2) follows from the definition of $T^\star$. ■

Harte proved that if $T^\star$ is injective and E, F are normed spaces, then T is a monomorphism, see [12, Th. 3.3.5.2]. The same conclusion can also be drawn for F metrizable.

Proposition 2 *Let E be a normed space and $T \in L(E,F)$ with $T^\star$ injective. Then $T^{-1} : T(E) \to E$ is sequentially continuous. In addition, if F is metrizable, then T is a monomorphism.*

PROOF. Suppose that $T^{-1} : T(E) \to E$ is not sequentially continuous. There exists a sequence $(x_n)_n \subset E$ such that $(Tx_n)_n \in c_0(F)$, but $(x_n)_n \notin c_0(E)$. Therefore there exists a subsequence $(x_{n_k})_k$ and $\varepsilon > 0$ such that $\|x_{n_k}\| \geq \varepsilon$ for all $k \in \mathbb{N}$. Define $(z_k)_k \in \ell_\infty(E)$ as $z_k := x_{n_k}/\|x_{n_k}\|, k \in \mathbb{N}$. Since $(Tx_{n_k})_k \in c_0(F)$, we get $(Tz_k)_k \in c_0(F)$. As $T^\star$ is injective we have $(z_k)_k \in c_0(E)$, a contradiction. If F is metrizable and T^{-1} is sequentially continuous, then it is continuous, and T is a monomorphism. ■

The assumption that F is metrizable cannot be dropped to obtain that T is a monomorphism as the next examples shows.

Example 2 (1) *Let T be the identity operator $T : (\ell_1, \|\cdot\|) \to (\ell_1, \sigma(\ell_1, \ell_\infty))$. It is bijective but it is not open because the norm topology is strictly finer than the weak topology on every infinite-dimensional Banach space. By Schur's lemma, e.g.* [15, 22.4(2)], *ℓ_1 has the property that all their weakly convergent sequences are also norm convergent. So that, T^{-1} is sequentially continuous but it is not continuous.*

(2) *There exists a linear partial differential operator with constant coefficients $P(D) = \sum_{|\alpha| \leq m} a_\alpha D^\alpha$, and an open set $\Omega \subset \mathbb{R}^n$ such that $P(D) : C^\infty(\Omega) \to C^\infty(\Omega)$ is surjective, but $P(D) : \mathcal{D}'(\Omega) \to \mathcal{D}'(\Omega)$ is not surjective. Such an example can be found in* [13, 14]. *It can be proved that if $P(D) : C^\infty(\Omega) \to C^\infty(\Omega)$ is surjective, then the operator $T := P(D)' : \mathcal{D}(\Omega) \to \mathcal{D}(\Omega)$ has a sequentially continuous inverse $T^{-1} : T(\mathcal{D}(\Omega)) \to \mathcal{D}(\Omega)$, so $T^\star$ is injective. Nevertheless, T cannot be a monomorphism because $T' := P(D) : \mathcal{D}'(\Omega) \to \mathcal{D}'(\Omega)$ would be surjective by the Hahn-Banach theorem. A contradiction.*

3 Surjectivity

An operator $T \in L(E, F)$ is surjective whenever T^∞ or T^0 are surjective. To deal with the converse we need to introduce the notion of lifting of bounded sets. $T \in L(E, F)$ is said to *lift bounded sets* if for every bounded set $B \subset F$ there is some bounded set $C \subset E$ such that $B \subset T(C)$. Quasinormable Fréchet spaces and their relevance for the lifting of bounded sets can be seen in [11], [17, Ch. 26] and [4]. Clearly, if T lifts bounded sets, then T and T^∞ are surjective. The surjectivity of T is not enough to get that T^∞ is also surjective.

Example 3 *There exists a Köthe echelon space $E = \lambda_1(A)$ which is a Montel space with quotient isomorphic to $F = \ell_1$; see* [15, Ex. 31.5] *or* [17, Ex. 27.21 and Prop. 27.22]. *The quotient mapping $q : E \to F$ does not lift bounded sets since the bounded sets of E are relatively compact. In this case q is surjective, but q^∞ is not surjective.*

If T^∞ is surjective, then $T^\star$ is also surjective. However, the converse sometimes fails.

Example 4 *Let X_0 be a non-complete normed space and let X be its norm-completion. Let T be the inclusion operator $T : X_0 \hookrightarrow X$. T is not surjective, nor T^∞, but T^* is by* [12, Th. 5.7.1], *since T is almost open.*

Finally, we study necessary conditions to ensure that T^0 is surjective.

Proposition 3 *If F is a metrizable l.c.s. and $T \in L(E, F)$ lifts bounded sets, then T^0 is surjective.*

PROOF. Fix $(y_n)_n \in c_0(F)$. Since F is metrizable there exists $(\alpha_n)_n \subset \mathbb{K}$ such that $\lim_n |\alpha_n| = \infty$ and $(\alpha_n y_n)_n \in \ell_\infty(F)$. Besides there is $C \subset E$ such that $(\alpha_n y_n)_n \subset T(C)$. So that, there is $(x'_n)_n$ with $Tx'_n = \alpha_n y_n, n \in \mathbb{N}$. Defining $x_n := x'_n/\alpha_n, n \in \mathbb{N}$, we have $(x_n)_n \in c_0(E)$ and $T^0(x_n)_n = (y_n)_n$. ■

Proposition 4 *Let E, F be metrizable l.c.s. If $T \in L(E, F)$ is surjective and open, then T^0 is surjective. In particular, a surjective operator T between Fréchet spaces satisfies that T^0 is surjective.*

PROOF. Let $(U_k)_k$ be an absolutely convex zero-neighbourhood basis in E. Clearly, $(T(U_k))_k$ is a zero-neighbourhood basis in F. If $(y_n)_n \in c_0(F)$, we can find a strictly increasing sequence $(n_k)_k \subset \mathbb{N}$, with $n_1 = 1$, such that $y_n \in T(U_k), n \geq n_{k+1}$. Define the sequence $(x_n)_n \subset E$ as follows: for every $1 \leq n < n_2$ we take $x_n \in E$ with $Tx_n = y_n$, and inductively, for every $n_k \leq n < n_{k+1}$ we take $x_n \in U_{k-1}$ with $Tx_n = y_n$. Clearly, $(x_n)_n \in c_0(E)$ and $T^0(x_n)_n = (y_n)_n$. ■

Example 5 *There exists a (DF) space, $F := \mathrm{ind}_n F_n$, which is also a complete (LB) space, with sequences converging to 0 in F which are not converging to 0 in any step F_n. Such a space is said to be not sequentially retractive. As an example take $F = (\lambda_1(A))'_b$, being the dual of a Köthe echelon space $\lambda_1(A)$ which is Fréchet-Montel, but it is not Fréchet-Schwartz* [17, Ej. 27.21]. *The space in Example* 3 *verifies these conditions. Consider the quotient $T : E \to F$, with $E := \oplus_{k=1}^{\infty} F_k$ defined as follows. If $x \in E$ is represented as $(x^k)_k$ with $x^k \in F_k$ for every $k \in \mathbb{N}$, then we define $Tx := \sum_{k=1}^{\infty} x^k$ for every $x = (x^k)_k$ in E. Take a sequence $(x_n)_n \in c_0(E)$. This is contained in $\oplus_{k=1}^{k_0} F_k$ for some $k_0 \in \mathbb{N}$, therefore $(Tx_n)_n \subset F_{k_0}$ and $(Tx_n)_n$ tends to zero in F_{k_0}. However, there exist sequences in $c_0(F)$ without preimage in $c_0(E)$.*

Example 6 *Let φ be the space of eventually null sequences in $\mathbb{K}$. This can be endowed with the sum norm $\|\cdot\|_1$, and with the euclidean norm, $\|\cdot\|_2$. Let $T : (\varphi, \|\cdot\|_1) \to (\varphi, \|\cdot\|_2)$ be the identity operator. This is clearly surjective and continuous, but it is not open. If it was the case, these norms $\|\cdot\|_1$ and $\|\cdot\|_2$ would coincide in a dense subspace of ℓ_1. This would imply $\ell_1 = \ell_2$. On the other hand, the operator $T^0 : c_0(\varphi, \|\cdot\|_1) \to c_0(\varphi, \|\cdot\|_2)$ is injective but it is not surjective: We define*

$$y_n := (0, \dots, 0, 1/(n+1), \dots, 1/(2n), 0, \dots), \qquad n \in \mathbb{N}.$$

Clearly, $y_n \in \varphi$ and $\|y_n\|_1 \geq \frac{1}{2}$ for every $n \in \mathbb{N}$, so $(y_n)_n \notin c_0(\varphi, \|\cdot\|_1)$, but $(y_n)_n \in c_0(\varphi, \|\cdot\|_2)$. Finally, using the injectivity of T^0, we obtain that there is no $(x_n)_n \in c_0(\varphi, \|\cdot\|_1)$ verifying also $T^0(x_n)_n = (y_n)_n$.

4 Homomorphisms

In this section we study conditions to state when T^∞, T^0 and $T^\star$ are homomorphisms, monomorphisms or open. It is easy to see that T is a homomorphism whenever T^∞ or T^0 is a homomorphism.

Theorem 2 (1) *If $T \in L(E, F)$ is a monomorphism, then T^∞ is a monomorphism.*

(2) *If $T \in L(E, F)$ is a homomorphism, then T^0 is.*

PROOF.

(1) We only need to prove that T^∞ is a homomorphism: For every $U \in \mathcal{U}_0(E)$ we have to find some $V \in \mathcal{U}_0(F)$ such that $(V^{\mathbb{N}} \cap \ell_\infty(F)) \cap T^\infty(\ell_\infty(E)) \subset T^\infty(U^{\mathbb{N}} \cap \ell_\infty(E))$. Given $U \in \mathcal{U}_0(E)$ there is $V \in \mathcal{U}_0(F)$ such that $V \cap T(E) \subset T(U)$. Consider $(y_n)_n \in V^{\mathbb{N}} \cap \ell_\infty(F)$ such that $Tx_n = y_n, n \in \mathbb{N}$, for a sequence $(x_n)_n \in \ell_\infty(E)$. The sequence $(x_n)_n$ belongs to $U^{\mathbb{N}} \cap \ell_\infty(E)$ because $(Tx_n)_n \subset V \cap T(E) \subset T(U)$ and T is injective.

(2) For every $U \in \mathcal{U}_0(E)$ we have to find some $V \in \mathcal{U}_0(F)$ such that $(V^{\mathbb{N}} \cap c_0(F)) \cap T^0(c_0(E)) \subset T^0(U^{\mathbb{N}} \cap c_0(E))$. Given $U \in \mathcal{U}_0(E)$ there exists $V \in \mathcal{U}_0(F)$ such that $V \cap T(E) \subset T(U)$. Consider $(y_n)_n \in V^{\mathbb{N}} \cap c_0(F)$ such that $Tx_n = y_n, n \in \mathbb{N}$, for a sequence $(x_n)_n \in c_0(E)$. There exists $n_0 \in \mathbb{N}$ such that $x_n \in U$ for every $n \geq n_0$. On the other hand there exists $z_n \in U$ such that $Tz_n = y_n$ for every $n \in \mathbb{N}$. Finally, consider the sequence $(w_n)_n$ defined as: $w_i = z_i$ for $1 \leq i < n_0$ and $w_i = x_i$ for $i \geq n_0$. This sequence belongs to $U^{\mathbb{N}} \cap c_0(E)$ and $T^0((w_n)_n) = (y_n)_n$. ■

Remark 1 *Let $T \in L(E, F)$ and T^∞ be a homomorphism. As $T^\infty(c_0(E)) \subset c_0(F)$ it follows that $T^\star : \ell_\infty(E)/c_0(E) \to \ell_\infty(F)/c_0(F)$ is a homomorphism.*

However, $T^\star$ can be a homomorphism without T^∞ being a homomorphism, c.f. see [12, Th. 4.7.4].

Example 7 *Let X, Y be normed spaces with Y complete and $T \in L(X, Y)$ be injective, surjective but not open. Neither T nor T^∞ are homomorphisms. However, T is almost open, and then $T^\star$ is open by* [12, Th. 3.4.5.2].

For T open, the fact that T^∞ is open was completely characterized by Dierolf and Bonet, using the bounded decomposition property (BDP), see [7]. Concerning quasinormability, there are results of De Wilde [9], Cholodovskiĭ [8], and Dierolf and Bonet [7, Prop. 1] that characterize the relation between surjectivity and openness for q^∞, see [7, p. 67]. The following result summarizes all these results.

Theorem 3 *Let Y be a separable Fréchet space. The following are equivalent:*

1. *Y quasinormable,*
2. *for every Fréchet space E containing Y, the operator q^∞ is a homomorphism, $q : E \to E/Y$ being the quotient map.*
3. *for every Fréchet space E containing Y, the operator q^∞ is surjective, $q : E \to E/Y$ being the quotient map.*

The foregoing conditions must be verified for all E. On the one hand, the Köthe echelon space in Example 3, has a quotient q isomorphic to ℓ_1. In this case q^∞ is not surjective, but it is a homomorphism. On the other hand, the operator q^∞, associated to a quotient q, can be surjective without being a homomorphism, as the next example shows. In this example we refer to the (DDC) for (DF) spaces, see [4] for more details about it.

Example 8 *Consider an (LB) space $F = \mathrm{ind}_n F_n$ that does not verify the (DDC). Köthe and Grothendieck introduced a Köthe matrix $A = (a_{i,j;k})_{(i,j)\in\mathbb{N}^2,k\in\mathbb{N}}$ defined as follows* [15, Sec. 31.7]*:*

$$a_{i,j;k} := \begin{cases} 1 & \text{for } i \geq k,\ i \in \mathbb{N}, \\ j & \text{for } i < k,\ i \in \mathbb{N}. \end{cases}$$

If we consider $F := \lambda_p(A)'_b$ for $1 < p < \infty$, then $\ell_\infty(F)$ is not barrelled. This is a complete regular (LB) *space without the* (DDC) [3]. *For a further treatment of this kind of examples, we refer to* [10, 4.7]. *Consider the quotient $T : E \to F$, where $E := \oplus_{k=1}^\infty F_k$, as we do in Example 5. This is a strict inductive limit of Banach spaces, and hence it verifies the* (DDC)*. By* [4, Th. 14] *we have that $\ell_\infty(E)$ is quasibarrelled. Since E is complete, then $\ell_\infty(E)$ is also barrelled. Clearly, T is continuous and surjective. Besides, both of them are* (LB) *spaces, so if we apply a proper version of the open mapping theorem,* [18, Th. 8.4.11] *or* [17, Th. 24.30]*, we conclude that T is open. On the other hand, $T^\infty : \ell_\infty(E) \to \ell_\infty(F)$ is continuous, and it is also surjective due to the regularity of F. However, T^∞ cannot be open because $\ell_\infty(E)$ is barrelled, and every quotient of a barrelled space is barrelled. A contradiction.*

Finally, we study the relevance of lifting bounded sets to determine that T^∞ is a homomorphism.

Theorem 4 *Let F be a metrizable l.c.s. If $T \in L(E, F)$ is an open operator lifting bounded sets, then T^∞ is open.*

PROOF. Fix $U \in \mathcal{U}_0(E)$. We can find $V_1 \in \mathcal{U}_0(F)$ such that $V_1 \subset T(U)$. Applying [17, Lemma 26.11] we can find $V_2 \in \mathcal{U}_0(F)$ verifying that for every $(y_n)_n \subset V_2$, there exists a bounded sequence $(x_n)_n \subset U$ such that $Tx_n = y_n$ for every $n \in \mathbb{N}$. If we define $V := V_1 \cap V_2$, then it can be seen $V^{\mathbb{N}} \cap \ell_\infty(F) \subset T^\infty(U^{\mathbb{N}} \cap \ell_\infty(E))$. ■

The hypothesis of lifting bounded sets is necessary. In Example 3, we have that q is open, but q^∞ is not surjective, so q^∞ cannot be open. Nevertheless, the hypothesis of lifting bounded sets is not necessary for the study of T^0: If E, F are metrizable l.c.s. and $T \in L(E, F)$ is open, then T^0 is open. However, this result is not true for non-metrizable (DF) spaces. The operator T in Example 5 is an open operator between (DF) spaces, but T^0 is not surjective so it cannot be open.

Acknowledgement. I thank José Bonet for helpful discussions and references concerning the results of this paper.

The author is supported in part by MEC and FEDER, Project MTM2004-02262 and Research Net MTM2006-26627-E.

References

[1] Abramovich Y. A., Aliprantis, C. D. and Polyrakis, I. A., (1996). Some remarks on surjective and bounded below operators, *Atti Sem. Mat. Fis. Univ. Modena*, **XLIV** 455–464.

[2] Berberian, S. K., (1974). *Lectures in functional analysis and operator theory*, Springer-Verlag, New York.

[3] Bierstedt K. D. and Bonet, J., (1988). Dual density conditions in (DF)-spaces I. *Results Math.*, **14**, 242–274.

[4] Bierstedt K. D. and Bonet, J., (2003). Some aspects of the modern theory of Fréchet spaces. *Rev. R. Acad. Cien. Serie A. Mat.*, **97**, 2, 159–188.

[5] Bonet, J. and Conejero, J. A., (2001). Duality for monomorphisms and almost open operators between locally convex spaces. *Bull. Soc. Roy. Sci. Liège*, **70**, 4-5-6, 183–193.

[6] Bonet, J. and Conejero, J. A., (2001). The sets of monomorphisms and of almost open operators between locally convex spaces. *Proc. Amer. Math. Soc.*, **129**, 3683–3690.

[7] Bonet, J. and Dierolf, S., (1990). Countable strict inductive limits and the bounded decomposition property. *Proc. R. Ir. Acad.*, **90A**, 1, 63–71.

[8] Cholodovskiĭ, V. E., (1976). On quasinormability of semi-metrizable topological vector spaces. *Funk. Analiz.*, **7**, 157–160.

[9] De Wilde, M., (1974). Sur le relèvement des parties bornées d'un quotient d'espaces vectoriels topologiques. *Bull. Soc. Roy. Sci. Liège*, **43**, 299–301.

[10] Dierolf, S., (1985). On spaces of continuous linear mappings between locally convex spaces. *Note Mat.*, **5**, 147–255.

[11] Dierolf, S., (1993). On the three-space problem and the lifting of bounded sets. *Collect. Math.*, **44**, 81–89.

[12] Harte, R., (1988). *Invertibility and singularity for bounded linear operators*. Marcel Dekker, New York and Basel.

[13] Hörmander, L., (1962). On the range of convolution operators. *Ann. of Math.*, **76**, 148–170.

[14] Kascic Jr., M. J. and Roth, B., (1970). A closed subspace of $\mathcal{D}(\Omega)$ which is not an (LF)-space. *Proc. Amer. Math. Soc.*, **24**, 801–802.

[15] Köthe, G., (1969). *Topological vector spaces, I*, Springer-Verlag, Berlin-Heidelberg-New York.

[16] Köthe, G., (1979). *Topological vector spaces, II*, Springer-Verlag, Berlin-Heidelberg-New York.

[17] Meise, R. and Vogt, D., (1997). *Introduction to functional analysis*, Oxford University Press, Oxford-New York.

[18] Pérez Carreras, P. and Bonet, J., (1987). *Barrelled locally convex spaces*, North-Holland Mathematics Studies, vol. 131, North-Holland, Amsterdam.

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